An Interesting result in ER Network

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Outline

- Part1: Generate an ER network with specific $< k >$
- Part2: For the generated ER network, calculate P_S by simulation and numerical method and do a comparison
- Part3: The interesting result in the evolution of ER network

Part 1: ER network

- $G(n, l)$ model consists of n nodes connected by l randomly placed edges

- $G(n, p)$ model consists of n nodes and edges linking each pair of nodes with probability p

Part 1: Generate ER network

Algorithm 1 Generate ER network

Require: N, p

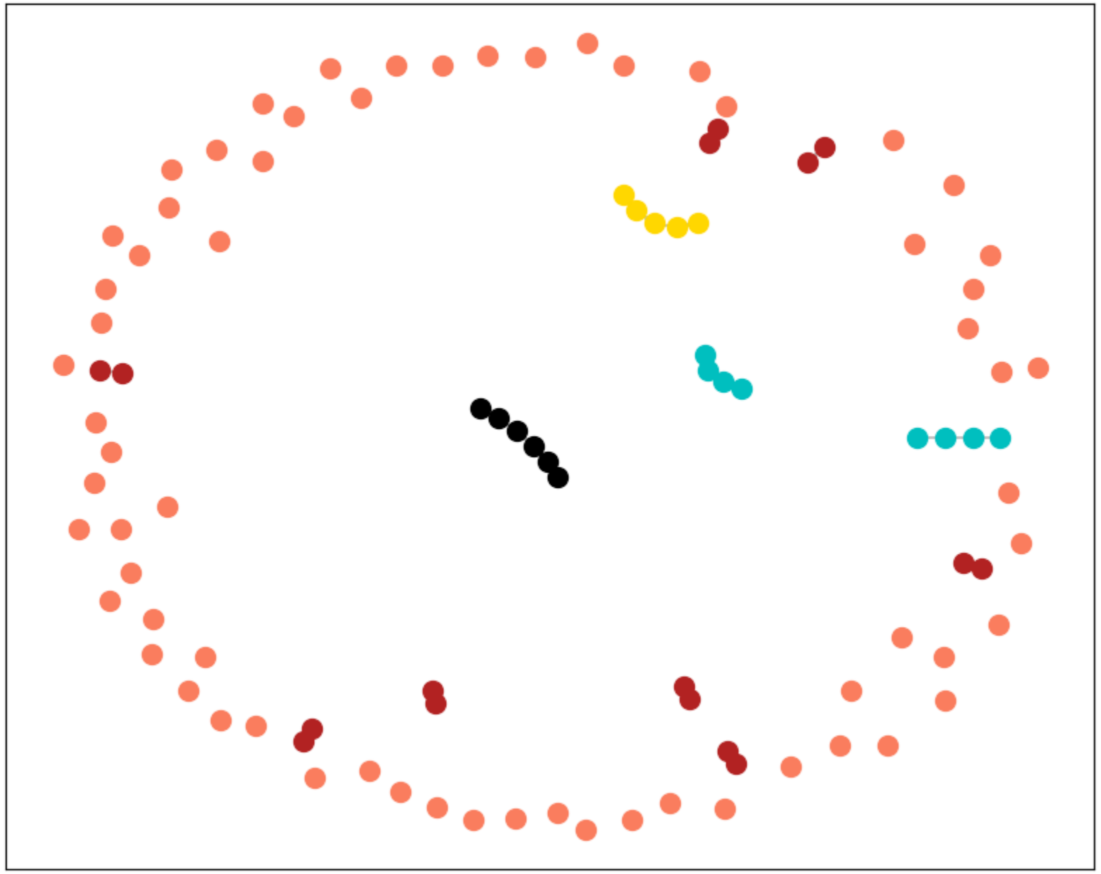
```
Create N nodes
for node i do
    for node j < i do
        generate random number u from U(0, 1)
        if u < p then
            add an edge between node i and j
        end if
    end for
end for
```

Ensure: network G

$N = 50, p = 0.6$

Part 2: obtain P_S by simulation

P_S: Randomly choose a node, the probability that the node belongs to a community of size s



$$\langle k \rangle = \frac{1}{n}\sum_{i=1}^{n} k_i = p(n-1)$$

Set $< k > = 0.5$

Generate ER networks from $G\left(100, \frac{0.5}{99}\right)$ 1000 times

Part 2: Obtain P_S by numerical method

$$P_s = \frac{1}{2\pi i} \oint \frac{H_0(z)}{z^{s+1}} \, \mathrm{d}z$$

definition

$$\downarrow z = e^{i\theta}$$

$$P_s = \frac{1}{2\pi} \int_0^{2\pi} \frac{H_0\left(\mathrm{e}^{i\theta}\right)}{\mathrm{e}^{i\theta s}} \, \mathrm{d}\theta$$

↓ split the interval

$$P_s \approx \frac{1}{2\pi} \sum_{k=0}^{N-1} \frac{H_0\left(\mathrm{e}^{i\frac{2k\pi}{N}}\right)}{\mathrm{e}^{i\frac{2k\pi s}{N}}} \times \frac{2\pi}{N}$$

$$G_0(x) = \sum_k p_k x_k = (1 - p + px)^{N-1}$$

$$G_1(x) = \frac{G_0'(x)}{G_0'(1)} = (1 - p + px)^{N-2}$$

$$\begin{cases} H_1(x) = xG_1(H_1(x)) \\ H_0(x) = xG_0(H_1(x)) \end{cases}$$

Part 2: result and conclusion

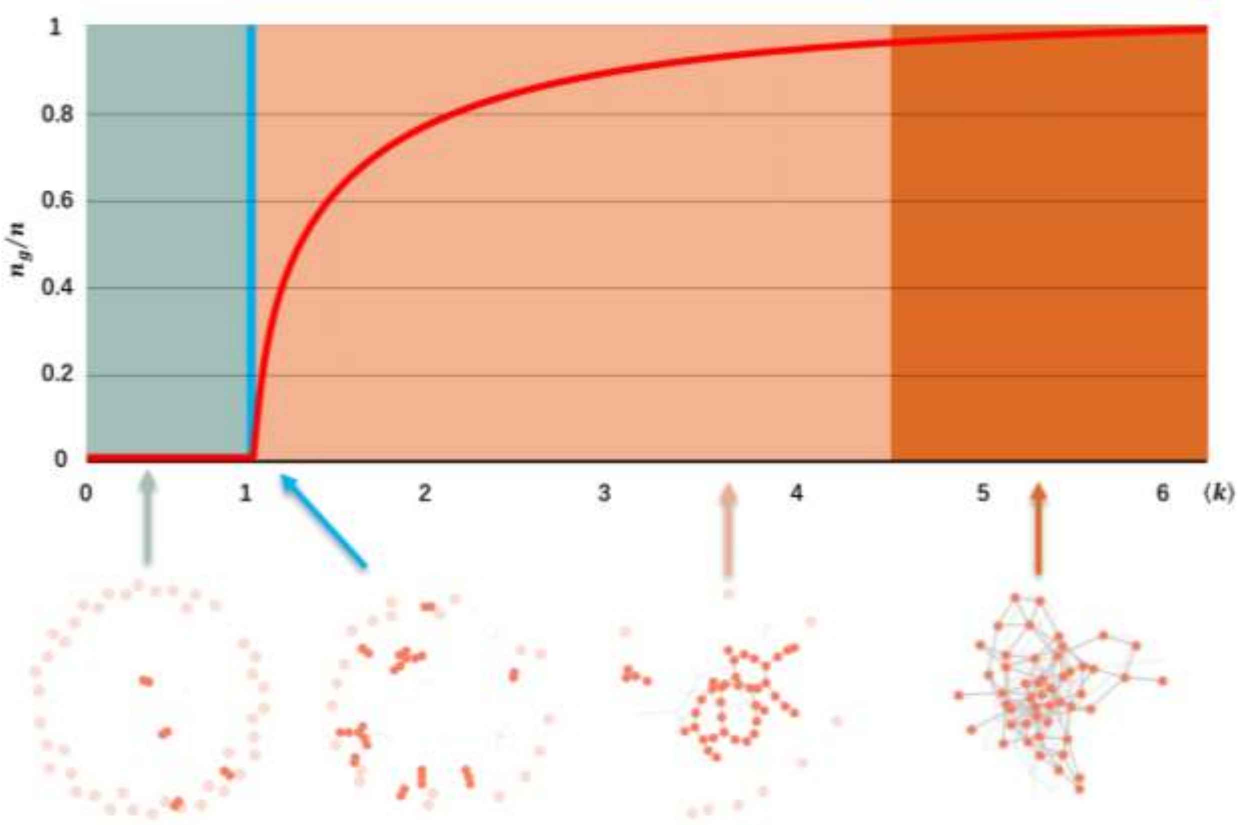
Size s	P_s from simulation	P_s from numerical calculation
1	0.6020	0.6088
2	0.1921	0.1853
3	0.0882	0.0838
4	0.0460	0.0448
5	0.0235	0.0262
6	0.0171	0.0163
7	0.0102	0.0106
8	0.0076	0.0071
9	0.0045	0.0048
10	0.0035	0.0034
11	0.0028	0.0024
12	0.0006	0.0017
13	0.0013	0.0012
14	0.0000	0.0009
15	0.0000	0.0007
16	0.0000	0.0005

(1)The results from two different methods are consistent;

(2)The results are closer when size s is small;

(3)When s is big, the numerical calculation method will give a non-zero result while the simulation will give 0.

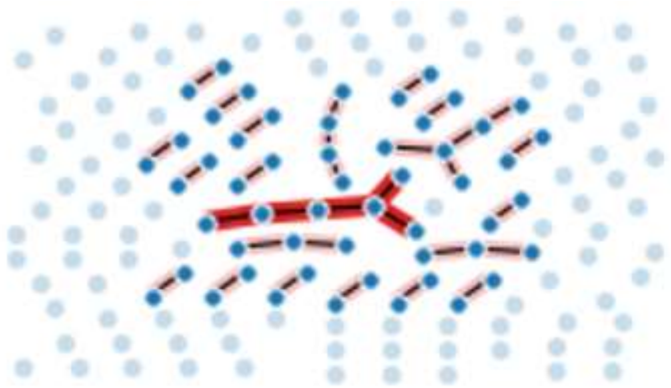
Part 3: The interesting result in the evolution of ER network



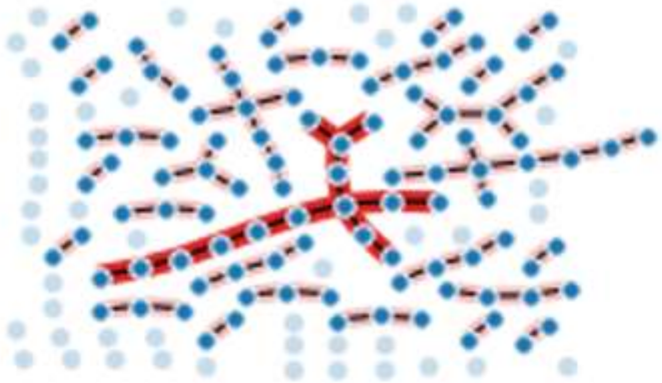
Let $p = \frac{1-\varepsilon}{n}$, where $\varepsilon > 0$ is a small enough constant and let $G \sim G(n, p)$. Then **w.h.p.** all connected components of G are of size at most $\frac{7}{\epsilon^2} ln n$.

Let $p = \frac{1+\varepsilon}{n}$, then **w.h.p.** G has a connected component with at least $\frac{\varepsilon n}{2}$ nodes.

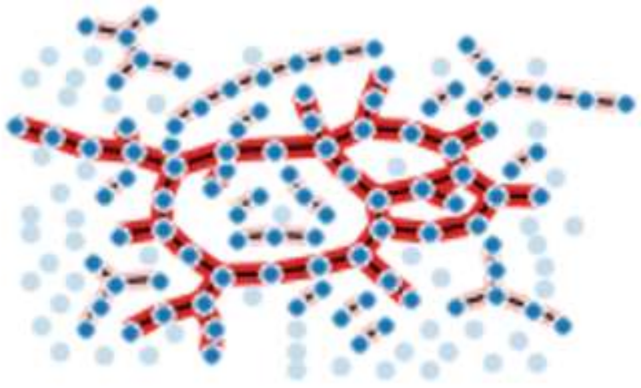
There exist a phase transition at p $= \frac{1}{n}$!



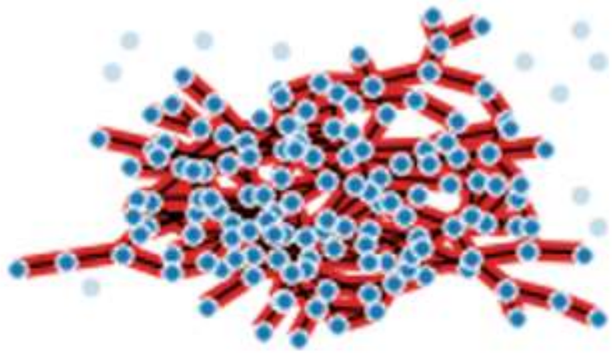
(1) $p = 0.003,\ n_g = 6, n_g/n = 0.04$



(2) $p = 0.006, n_g = 15, n_g/n = 0.1$



(3) $p = 0.008, n_g = 45, n_g/n = 0.3$



(4) $p = 0.015, n_g = 139,\ n_g/n = 0.93$

a simulation of ER network evolution with $n = 150$ $(p = \frac{1}{150} \approx 0.007)$

Thanks for listening!